CALCULATION OF FLAME WAVELENGTH IN VIBRATORY COMBUSTION IN TUBES

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A calculation is made of the wavelength at the surface of a flame in vibratory combustion in a tube, allowing for preheating of the fuel mixture through heat conduction ahead of the ignition front.

In a previous paper [1], the author represented the mechanism of vibratory combustion of gas mixtures in tubes as resulting from interaction of the self-oscillations of the gas column (fuel mixture-combustion products) in the tube with oscillations produced by hydrodynamic instability of the flame front. From the positions of resonance coincidence of frequencies of the above oscillations we have derived a simple theoretical formula for determining the wavelength λ of the disturbance prevailing at the flame front, which was assumed to be an oblique expansion shock:

$$\lambda = v_1 \cos \alpha / \omega_n . \tag{1}$$

Here v_1 is the rate of propagation of burning in the direction of the tube axis, α is the mean angle of inclination of the flame surface to the tube axis, and ω_n is the frequency of the self-oscillations of the gas in the tube. The experiments conducted in [1] on burning of CO-air mixtures in tubes with flame propagation from an open end to a closed one gave values for the wavelength λ at the ignition front approximately three times larger than given by (1). The chief cause of lack of agreement between theory and experiment lies in the fact that in the experiments the mean rate of propagation of the flame relative to the tube was measured, this being the velocity of the burning zone along the tube axis relative to the original fuel mixture. This, as will now be shown, leads to a reduction of the true value of the burning rate relative to the original gas. Indeed, in the one-dimensional case the stationary flame is a zone of a certain width L, lying between $x = -L$ and $x = 0$, if the coordinate system is related to the flame in such a way that the latter is at rest, while the gas flows into it and out of it. Fuel mixture with parameters ρ_1, v_1, T_1 flowing into this zone from the left turns into combustion products with parameters ρ_2, v_2, T_2 flowing out into the region $x > 0$. Then, as it enters the flame zone, the mixture is first heated due to the heat flux from the incandescent combustion products, and after it reaches a high enough temperature, an energetic chemical reaction rapidly takes place to transform it into combustion products. Because of the strong dependence of the rate of the chemical reaction on temperature, the chemical reaction zone is a relatively small part of the flame width. Then, neglecting the size of the chemical transformation zone, we may represent the flame [2] by a thermal wave in the initial mixture ahead of the surface $x = 0$ heated to temperature T_2. The result is the following boundary value problem in the theory of steady heat conduction [2]:

$$v_1 \frac{dT}{dx} = \chi_1 \frac{d^2T}{dx^2}, \; x=0 \text{ when } T=T_2, \; x=-\infty \text{ when } T=T_1$$

with solution

$$T = T_1 + (T_2 - T_1) \exp\left(\frac{v_1}{\chi_1} x\right). \tag{2}$$

The width of the flame is understood [3] to be only that of the section of effective temperature increase, calculated from dimensional considerations in the form $L = \chi_1/v_1$. Therefore, using the discontinuous flame front model in the theoretical calculations in which (1) was obtained, we must take into account that, because of the low normal burning rate, the flame is able to heat the original mixture ahead of it. In fact, therefore, it is propagating in a gas with higher temperature T_*. In order to evaluate this heating, we shall assume T_* to be the mean integral from (2) in the section of unit width of flame zone L directly ahead of it, i. e., in the range $(x = -2L)-(x = -L)$. We then have

$$T_* = T_1 + (T_2 - T_1)\left(1 - \frac{1}{e}\right)\frac{1}{e}. \tag{3}$$

The gas directly in front of the flame expands and is set in motion when heated. For combustion propagating from the open to the closed end of a tube, as occurred in our tests [1], the above motion of the fuel mixture can only be toward the flame (toward the open end of the tube). This produces an increase of the true value of v_1 in (1) up to v_*, i. e., up to the velocity of propagation of the flame relative to the original mixture, compared to the velocity of propagation of the combustion front relative to the tube measured in the experiments. To find this true velocity v_*, we use (3) in the equations of continuity and state for a slow combustion process

$$\rho_* v_* = \rho_1 v_1, \quad \rho_* T_* = \rho_1 T_1.$$

Hence at once we obtain

$$v_* = v_1 [1 + (T_2/T_1 - 1)(e-1)/e^2], \tag{4}$$

which gives, for example, $v_* = 2.2 v_1$ for $T_2/T_1 = 6$. Therefore, substituting v_* in (1) in place of v_1, we have

$$\lambda = \frac{v_1 \cos\alpha}{\omega_n} \left[1 + \left(\frac{T_2}{T_1} - 1\right) \frac{e-1}{e^2}\right] \qquad (5)$$

which already gives very good agreement with the experimental relation. The discrepancy between (5) and the experimental results [1] is about 20–25% instead of a factor of 3.

NOTATION

λ—wavelength of disturbance at flame front; ω_n—frequency of self-oscillations of gas in tube; v_1—mean burning rate; ρ_1, T_1—mean density and temperature of original mixture; ρ_2, T_2—density and temperature of combustion products; v_*—true burning rate; ρ_*, T_*—true density and temperature immediately ahead of flame; χ_1—diffusivity of original mixture; L—flame width; α—mean angle of inclination of flame to tube axis.

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